SHOCK ATTENUATION OF SPORTS SURFACES

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ABSTRACT: Shock attenuation is an important property of sports surfaces, especially when impact carries a risk of severe injury. This paper explores the effects of non-linear cushioning material properties and contact geometries on surface shock attenuation performance, using a power-law force-displacement model to describe a broad class of non-linearities. Analysis shows that the maximum displacement of the surface during an impact and the non-linearity of the force-displacement relationship are important determinants of cushioning performance. Typical sports surfaces tend to stiffen when compressed, a material non-linearity that increases the peak acceleration of an impact but reduces impact severity criteria that depend on the average acceleration. Under many conditions and especially when surface thickness is limited, materials that soften or collapse when compressed provide better shock attenuation.

INTRODUCTION

Injuries caused by collisions between athletes and hard surfaces are a common occurrence in sports. Consequently, the surfaces in many sports installations are required to attenuate impact. Artificial turf fields, sports hall floors, running tracks, tennis courts, and gymnastic crash mats are examples of surfaces that are required to exceed minimum shock attenuation criteria established by sports governing bodies and other agencies. Frequently, goal posts, walls and other vertical surfaces are also cushioned to reduce the risk of injury. Shock attenuation is especially important when the risk of head injury or other severe injury is non-trivial.

The design and development of shock attenuating surfaces requires some understanding of how material properties and impact conditions interact. In this regard, mathematical models of impact provide a means of developing understanding and find use as practical tools in the design of new surfaces. McMahon and Greene (1979), for example, used a damped mass-spring system to analyse and optimize the properties of an indoor running track. Hertz’s contact theory (e.g. Johnson, 1985, Ch.4) has also found some application to sports surfaces, but the validity of Hertzian models breaks down in the face of significant non-linearities (Pavis et al, 1983).

This paper describes an investigation of how non-linear elasticity and contact geometries affect the performance of shock attenuating surfaces, using a non-linear force-displacement model. The application of the model and its results to the design of shock-attenuating surfaces is also demonstrated.
NON-LINEAR IMPACTS

The impact between an athlete’s head and a sports surface is fundamentally non-linear. Even if the surface were perfectly, linearly elastic and the head perfectly rigid, the geometry of the contacting surfaces would introduce non-linearities. For Hertzian contact between a rigid spherical impactor of mass \( m \) and a flat, linearly elastic half-space, the force \( (F) \)-displacement \( (x) \) function is non-linear and of the form

\[
F = m \frac{d^2 x}{dt^2} = k x^{3/2}
\]

where the stiffness constant \( k \) depends on the surface’s elastic modulus and the geometry of the impactor. Real sports surfaces are not ideally linearly elastic. Non-linear material properties, densification and energy dissipation all contribute to force-displacement behavior that is non-Hertzian. Simple forms of non-linear behaviour can be described by a more general force-displacement relationship:

\[
F = m \frac{d^2 x}{dt^2} = k x^n
\]

The non-linearity coefficient, \( n \geq 0 \), depends on material properties and contact geometry. The model encompasses linear impacts \( (n = 1) \), Hertzian contact \( (n = 3/2) \) as well as materials that stiffen \( (n > 1) \) or soften \( (n < 1) \) when compressed.

THEORETICAL DEVELOPMENT

Consider an impact in which a mass \( m \) collides with a surface at velocity \( v_0 \) and obeys the force-displacement relation of equation (2). Assuming energy is conserved,

\[
\frac{1}{2} v_0^2 \left( \frac{dx}{dt} \right)^2 = \frac{1}{(n+1) m} k x^{n+1}
\]

The maximum displacement of the surface, \( x^* \), occurs when \( dx/dt = 0 \). Hence

\[
x^* = \left[ \frac{k}{n+1} \left( \frac{1}{2} m v_0^2 \right)^{\frac{1}{n+1}} \right]^{\frac{1}{n+1}}
\]

The peak acceleration expressed in gravitational \( (g) \) units is then given by

\[
g_{\text{max}} = \frac{1}{g} \left( \frac{d^2 x}{dt^2} \right)_{\text{max}} = \frac{1}{mg} k \left( x^* \right)^n = \frac{1}{g} \frac{k}{m} \left[ \frac{(n+1) v_0^2}{2} \right]^n
\]

The elapsed time to maximum displacement, \( t^* \), is obtained by integration:

\[
t^* = \frac{x^*}{v_0} \int_0^1 \frac{d(x/x^*)}{\sqrt{1-(x/x^*)^{n+1}}} = \frac{x^*}{v_0} \frac{\Gamma \left( 1 + \frac{1}{n+1} \right)}{\Gamma \left( \frac{1}{2} + \frac{1}{n+1} \right)}
\]

Figure 1 shows dimensionless force-displacement, displacement-time and acceleration-time curves for some different values of \( n \), determined by numerical integration. As Figure 1(c) shows, the displacement-time curve is approximately sinusoidal for \( n < 3.0 \). The differential equation (2) is transcendental but numerical
analysis shows that the displacement-time function is well approximated by

\[ x = x^* \left[ \sin \left( \frac{\pi}{2} \frac{t}{t^*} \right) \right]^{1+\frac{(n-1)}{2n}} \]  

and the acceleration-time curve by

\[ \frac{1}{g} \frac{d^2x}{dt^2} = g_{\text{max}} \left[ \sin \left( \frac{\pi}{2} \frac{t}{t^*} \right) \right]^n \left( 1+\frac{(n-1)}{2n} \right) \]  

(8)

The power-law model is able to mimic simple non-linear behaviour in a variety of sports surfacing materials. For example, Figure 2 shows compressive stress-strain curves for three surfaces: an in-filled artificial turf product, a rubber playground surface and a gymnastic crash mat. While the crash mat was approximately linear up to 60% strain, the power-law model gives a substantially better fit to data from the other surfaces than the assumption that the surface is a linear elastic half space.

Figure 3a shows the acceleration response of the artificial turf product to an ASTM F-1936 impact test. With this protocol, a 9.1 kg, 12.8 cm diameter, cylindrical missile with a flat circular face is dropped onto the surface with an impact velocity of 3.5 m s\(^{-1}\). Since a cylindrical punch penetrating a linearly elastic half space has a

![Fig. 1](https://via.placeholder.com/150)

**Fig. 1** Effects of the non-linearity coefficient, \(n\), on (a) force-displacement, (b) acceleration-time and (c) displacement-time curves of an impact for \(n = 0.5 (\bigcirc), 1.0 (\square), 1.5 (\triangle), \text{ and } 5.0 (\biglozenge)\).

![Fig. 2](https://via.placeholder.com/150)

**Fig. 2** Compressive stress-strain curves of three sports surfacing materials.
linear force-displacement relationship (Johnson, 1985; Ch.3), the test method geometry does not introduce new non-linearities. The response of the playground surface sample to an impact with a spherical headform is also shown (Figure 3b). In this case, the response is a function of both the material non-linearity and the Hertzian contact geometry \( n = 1.5 \). In both cases a model of the form of (8), assuming \( n \) is the product of the stress-strain curve non-linearity (Figure 2) and the contact non-linearity, provides a better approximation of the data than linear or Hertzian models.

**OPTIMISATION OF CUSHIONING PERFORMANCE**

The general problem in sports surface shock attenuation is to manipulate surface properties in a way that reduces or minimises the severity of a defined impact event. Commonly used impact severity criteria include peak acceleration \( (g_{\text{max}}) \) and empirical indices like the Severity Index \( (SI) \) and Head Injury Criterion \( (HIC) \).

Thick layers of compliant materials solve many cushioning problems. They also increase the cost and reduce the portability of a surface installation and may adversely affect running speed, ball bounce, and other performance factors. Consequently, the maximum displacement of a surface is an important constraint on surface designs. If displacement is constrained to a maximum value \( x^* = x_{\text{max}} \), the peak acceleration is

\[
g_{\text{max}} = \frac{(n + 1) \, v_0^2}{2g \, x_{\text{max}}} \tag{9}
\]

Equation (9) shows that, for a given displacement constraint, lower values of \( n \) produce lower \( g_{\text{max}} \) scores. In the limiting case, \( g_{\text{max}} \) is a minimum when \( n = 0 \) and the acceleration-time curve is a rectangular pulse. While the theoretical minimum case may be neither practical nor desirable when other impact criteria are considered, this result suggests that more efficient shock attenuating surfaces can be made from materials or structures that soften or buckle when compressed.

The risk of severe traumatic head injury is not only a function of the peak impact shock, but also of the time to which the head is exposed to acceleration. The head can tolerate relatively long exposure to low accelerations but only brief exposure to higher ones. The Head Injury Criterion \( (HIC) \) (Versace, 1971; Lockett, 1985) is a commonly used, empirical index of impact severity, based on experimental studies of acceleration tolerance using cadavers, animals and human volunteers.
\[ HIC = (t_f - t_0) \left[ \frac{1}{(t_f - t_0)^4} \int_{t_0}^{t_f} \frac{1}{g} \frac{d^2 x}{dt^2} \right]^{2.5} \]  

(10)

where the HIC time interval \((t_f - t_0)\) maximises the HIC score. A HIC score of 1000 represents a threshold above which the risk of fatal or severe traumatic head injury is unacceptable. When \((t_f - t_0)\) occupies the entire contact duration, \(2t^*\), of an impact conforming to (8) with an average acceleration \(g_{avg}\), the HIC score is approximated by

\[ HIC = 2t^* (g_{avg})^{\frac{3}{5}} = f(n, x^* \frac{\gamma}{2}, \nu_0) \]  

(11)

and minimised by maximizing both \(n\) and \(x^*\). Thus HIC is minimised when surface properties are such that the impact energy is just absorbed at the maximum allowable compression of the surface.

**MATERIAL PROPERTY SELECTION**

Since \(g_{max}\) is minimised by low-\(n\) material properties and HIC is minimised by high-\(n\) material properties, the apparent performance of a surface depends on the impact severity criterion used. However, any shock attenuation criterion is ultimately constrained by the maximum displacement of the surface and hence by its thickness. Within this constraint, \(g_{max}\) and HIC are both reduced by low-\(n\) properties and the range of material properties that meet specific shock attenuation goals is more limited.

Alternative material solutions that meet particular shock attenuation goals can be determined by mapping impact severity criteria as functions of material properties. Figure 4 is an example that shows \(g_{max}\) and HIC scores for a 150 J impact of a 5 kg spherical headform. The lowest feasible HIC and \(g_{max}\) scores are shown as functions of the non-linearity coefficient, \(n\) and the maximum allowable displacement, \(x_{max}\).

![Figure 4](image-url)  

**Fig 4**: Optimal peak shock (\(g_{max}\)) and Head Injury Criterion (HIC) scores as a function of surface material thickness and non-linearity for a spherical headform with an impact energy of 150 J.
With maximum displacement or surface thickness as a constraint, peak acceleration \( g_{\text{max}} \) is sensitive to both the thickness and the non-linearity of the surface, while HIC scores are most sensitive to the available displacement. Increasing the allowable displacement reduces both \( g_{\text{max}} \) and HIC scores. Increasing the non-linearity coefficient increases \( g_{\text{max}} \) but has less effect on HIC unless \( n \) has a relatively low value. Because of these differing sensitivities, “high-\( n \)” surfaces are less likely to meet \( g_{\text{max}} \) criteria, while “low-\( n \)” surfaces are less likely to meet HIC criteria.

Typical sports surfaces have non-linear properties with \( n > 1 \), i.e. they tend to stiffen when compressed. However, when surface thickness is limited, there is some performance advantage to materials with low-\( n \) behaviour. As Figure 5 shows, the thinnest surfacing solution is generally the one with the lowest attainable value of \( n \).

**EXAMPLE APPLICATION: PLAYGROUND SURFACING**

In North America and elsewhere, playground surfaces are rated according to the maximum “critical fall height” from which the impact of a 5 kg headform meets the criteria \( g_{\text{max}} < 200 \text{g} \) and HIC < 1000 (ASTM F1292). In the USA, play surfaces are also required to be wheelchair accessible, which constrains surface compliance.

A typical unitary playground surface consists of a layer of granular or shredded recycled rubber bound together with polyurethane and covered with an EPDM rubber wearcourse. The performance of this system is directly related to the thickness and hence to the cost of the rubber layers. An 8 cm thick surface of this type fails the HIC<1000 criterion at a critical fall height of about 2m (Figure 5b).

A new surface design takes advantage low-\( n \) material properties. Replacing the granular rubber layer of the conventional surface with a thermoformed structure that buckles under load (Figure 5a) not only reduces the non-linearity coefficient of the surface but also increases the maximum displacement available before the surface bottoms out. The new design changes the performance potential of the surface from point A in Figure 4 to point B. As Figure 5b shows, the actual gains in shock attenuation performance are also significant. The low-\( n \) surface is also relatively stiff at low strains, a property that makes it more suitable for pedestrians and wheelchairs.

![Figure 5](image)

*Fig 5* (a) Thermoplastic structure used to create low-\( n \) properties in a shock attenuating playground surface (b) ASTM F-1292 HIC scores of the low-\( n \) surface compared with a conventional surface of the same thickness.
CONCLUSIONS

Non-linear behaviour has significant effects on the shock attenuation of sports surfaces. For many purposes, a power-law force-displacement model provides an adequate empirical description of surface non-linearities, including those due to material properties and contact interface geometry.

The model has value as a precursor and sometimes as a substitute for finite element modeling and prototyping. It also provides a useful engineering tool for optimisation across a wide range of material properties and for calculating limits on shock attenuation performance potential.

Analysis shows that surfaces that stiffen under load tend to produce higher peak accelerations during impact but lower average accelerations than those with linear properties. Conversely, surfaces that soften when compressed have relatively low peak accelerations and higher average accelerations. Optimal cushioning properties are therefore dependant upon the criterion used to evaluate the severity of an impact. While peak acceleration ($g_{\text{max}}$) is sensitive to both the thickness and the non-linearity of the surface, measures that depend on the integral of the acceleration-time pulse (e.g. $g_{\text{avg}}$ and HIC) are most sensitive to the available displacement in the surface.

Material properties that maximise the displacement of the surface during an impact yield the best shock attenuation. When maximum displacement of the surface is limited (by thickness constraints, for example), materials or structures that soften when compressed have better shock attenuation than those that stiffen. In general, the thinnest surface that meets a given shock attenuation criterion is the one with the lowest possible non-linearity coefficient.

REFERENCES


